

RESEARCH MEMORANDUM

A CASCADE—GENERAL-MOMENTUM THEORY OF OPERATION OF A
SUPERSONIC PROPELLER ANNULUS

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SUMMARY

A cascade—general-momentum theory method for calculating the operating conditions of a supersonic propeller annulus throughout the flight Mach number range is presented. The introduction of an infinite two-dimensional supersonic cascade as the method of power absorption permits the consideration of such effects as drag due to lift and thickness, shock interference, and solidity and appears useful in studying general trends of supersonic propeller operation. For simplicity in this presentation, sections with zero thickness and drag due to lift only are considered.

General flow patterns about the cascade and adjustments to free-stream conditions are discussed. Representative subsonic, transonic, and supersonic solutions are given.

INTRODUCTION

The simple compressible axial-momentum theory of reference 1 was derived in an attempt to develop a rational compressible-flow propeller theory. Solutions to the flow equations as presented in reference 1 that were acceptable from physical considerations were not found whenever the stream Mach number was greater than 1.0 or whenever power loading in excess of the amount required to induce an inflow Mach number of 1.0 was used. Because the simple momentum theory is not concerned with details of flow about the actual propeller blade, this difficulty could not be resolved.

In this paper the actuator disk is replaced by an infinite cascade representing an annulus of a supersonic propeller. By means of this substitution, a satisfactory physical picture of the flow phenomena is obtained.

The purpose of this paper is to present the basic considerations and general method of calculation of the cascade—general-momentum theory together with representative solutions in the transonic forward-speed range. As an incidental result, a logical method of extending the simple momentum theory is also indicated.

SYMBOLS

B	number of blades
b	blade chord, ft
D	propeller diameter, ft
F	force on blade element, lb/unit radial distance
J	advance ratio, V_0/nD
M	Mach number
m	mass flow, slugs/sec/unit radial distance
n	rotational speed, rps
P	power, ft-lb/sec/unit radial distance or hp/unit radial distance
P_c	power-disk loading coefficient, $\frac{P/S}{\rho_0 V_0^3/2}$
p	static pressure, lb/sq ft
Q	torque, ft-lb/unit radial distance
r	radius to a blade element, ft
S	blade spacing, b/σ , ft
T	thrust, lb/unit radial distance
U	translational velocity of cascade, ft/sec
V	velocity, ft/sec
W	flow velocity relative to cascade, ft/sec

x	fraction of propeller tip radius, $2r/D$
L.E.	plane of blade leading edges in cascade
T.E.	plane of blade trailing edges in cascade
P/S	power loading, hp/unit frontal area
0	initial stream conditions
3	final wake conditions
1,2	slipstream locations of reference 1, correspond in present theory to L.E. and T.E., respectively
α	angle of attack, $\beta_c - \phi_R$, deg
β	blade angle, deg
γ	ratio of specific heats, 1.4
η	efficiency
ρ	density, slugs/cu ft
σ	solidity, $\frac{Bb}{\pi x D}$
ϕ	angle between relative velocity W and YZ-plane
X,Y,Z	coordinate axes

Subscripts:

a	axial
c	cascade
R	local flow condition at blade L.E. relative to cascade
R'	local flow condition at blade T.E. relative to cascade
t	tangential, transverse
x	propeller radial station at x
LE	average condition along blade leading edges

TE average condition along blade trailing edges
0 free stream
3 final wake

DEVELOPMENT OF THEORY

General Considerations

The cascade—general-momentum theory developed herein differs from the simple axial-momentum theory of reference 1 in two important aspects as follows: (a) The actuator disk concept is replaced by an infinite two-dimensional cascade, and (b) the flow velocity has significant components in the tangential direction; thus, slipstream rotation is permitted.

The two-dimensional cascade is generated as shown in figure 1 where the span of the airfoils is taken to be unity in the radial direction and the distance S between airfoils (measured from leading edge) is b/σ .

The cascade may now be introduced into the idealized flow pattern as shown in figure 2 where the mean radius of the flow pattern is assumed constant. In this figure the reference axes are defined as follows:

X flight direction
Y direction of rotation
Z radial direction

The bounded region in figure 2 is described in the following manner:

(a) The "ends" of the region at stations 0 and 3 are composed of planes normal to the flight direction.

(b) Between stations 0 and L.E., the "sides" of the region are composed of parallel planes passing through the leading edges of two adjacent airfoils and extending forward in the flight direction. Because of the infinite extent of the generated cascade in the Y-direction, the net forces acting on and the net change in momentum across these boundaries become zero and need not be considered in the mathematical treatment.

(c) Between stations T.E. and 3, the "sides" are similarly composed of parallel planes.

(d) Between stations L.E. and T.E., the "sides" are composed of the upper surface of one airfoil and the lower surface of the adjacent airfoil. The forces on these surfaces represent the total force acting on each airfoil.

(e) Between stations L.E. and T.E., the "top" and "bottom" are composed of parallel planes separated by a unit radial distance.

(f) Between stations 0 and L.E. and T.E. and 3, the "top" and "bottom" are composed of surfaces which are not necessarily parallel but may converge or diverge in order for the flow to adjust to free-stream pressure.

In the region between stations L.E. and T.E., the problem consists only of the flow within an infinite two-dimensional cascade. In the regions between stations 0 and L.E. and stations T.E. and 3, the solution follows that of the general-momentum theory.

When a propeller annulus is represented by an infinite two-dimensional cascade, the assumption is made that any blade element is influenced only by other blade elements at the same radius. Radial variations in velocity and Mach number existing in a propeller and the helical path taken by an actual blade element are ignored. These factors cause a given blade element to be influenced both by other elements of the same blade and by elements of other blades at different radii. These mutual interference effects are left for future study.

The assumption of independence of annulus operation requires that the mean radius of the flow remain constant throughout. The effects of a constant mean radius on power, thrust, and efficiency are insignificant.

Because the method of attack presented is not direct, it is necessary to consider the problem in the inverse manner by obtaining a local flow pattern about the cascade and then proceeding to a consistent set of flight conditions.

Two-Dimensional Cascade

In the method of this paper, the flow conditions about an infinite two-dimensional cascade are first determined. The flow patterns are completely specified by the cascade geometry, by the flow conditions in the immediate vicinity of the leading edge of the airfoils designated as reference values by the subscript R , and by the pressure immediately behind the trailing edge designated by the subscript R' . The direction of the reference flow with respect to the blade determines the angle of attack α .

In order to simplify study of the flow about an infinite two-dimensional cascade, the assumption is made that the sections are frictionless flat plates of zero thickness. Because of the difficulties in handling mixed flow, the analysis is restricted to those cases where the flow relative to the blades is everywhere supersonic.

Based on two-dimensional supersonic-flow theory, six general types of flow pattern for a thrusting annulus appear possible and are illustrated in figures 3 to 7, in which expansion fans are indicated by dashed lines and shock waves by solid lines. The conditions under which these patterns exist are discussed in the following paragraphs. It will be noted that the type of pattern is intimately related to the axial components of M_R (X- or axial Mach number component of the flow at the blade leading edge). Observe that the axial component of M_R is different from the flight Mach number whenever induced inflow is present.

Subsonic, interference-free pattern (fig. 3).- When the X- or axial Mach number component of the flow at the blade leading edge (axial component of M_R) is sufficiently subsonic, all shocks and expansions from the blade upper surface will pass ahead of the following blades and the pressure immediately behind the trailing edge is for all practical purposes equal to the pressure immediately ahead of the leading edge. It will be noted that each blade operates essentially independent of other blades.

Subsonic, shock interference pattern (fig. 4).- For an axial component of M_R approaching 1.0, the trailing-edge shock is intercepted and reflected by the following blade. (See fig. 4.) When this phenomenon occurs all wave patterns originating at the leading edge disappear because the air approaches the blade leading edges at zero angle of attack α . The portion of the blades ahead of the reflected shock produces no lift; the induced flow required to bring the air into the blading at zero angle of attack is generated by the working parts of the blades behind the reflected shock. A discussion of a mechanism for maintaining this zero-angle-of-attack condition is given in reference 2.

The strength and location of the trailing-edge shock is fixed by the pressure immediately behind the trailing edge.

Sonic, shock-expansion interference patterns (fig. 5).- When the axial component of M_R is 1.0, the angle of attack α may be greater than zero and the patterns of figure 5 are possible. The angle of attack determines the amount of leading-edge expansion that interferes with the following blades. If the pressure immediately behind the trailing edge is assumed to be sufficiently higher than the pressure at the leading edge, an interfering shock will exist as shown in figure 5(a). The lowest pressure possible at the trailing edge for the pattern of figure 5(a) occurs when the trailing-edge shock intersects the trailing edge of the following blade. The pattern of figure 5(b) occurs when no

trailing-edge shock is present and in this case the trailing-edge pressure is equal to the pressure on the upper surface of the blade.

Although the patterns of figures 5(a) and 5(b) have identical entering conditions, they could correspond to very different operating conditions. The pattern of figure 5(b) represents operation slightly below sonic flight Mach number with relatively low power. The pattern of figure 5(a) is typical of operation at a lower flight Mach number with relatively higher power.

Supersonic, expansion-interference pattern (fig. 6).- When the axial component of M_R is slightly greater than 1.0, the pattern of figure 6 is possible. The leading-edge expansion may be completely or partially reflected by the following blade and will determine the pressure in the immediate vicinity of the blade trailing edge.

Supersonic, interference-free pattern (fig. 7).- When the axial component of M_R is sufficiently supersonic, all disturbances created by any blade pass behind all other blades; the pressure in the immediate vicinity of the trailing edge equals that at the leading edge. In this type of flow pattern, all blades operate exactly as isolated airfoils.

To this point in the solution of the general problem the cascade representation is sufficient to define the local flow pattern, thrust, and torque force of the corresponding propeller annulus. It should be observed that interference may cause significant changes in the blade forces. Detailed calculation of the effect of this interference on thrust, power, and efficiency are left for future study.

General Momentum Theory

With conditions at stations L.E. and T.E. established by the cascade approach, determination of appropriate uniform flight or free-stream conditions is made through use of the general-momentum theory. Details of the complete shock-expansion patterns are not necessary since a physical flow-averaging process is present in which all expansion fans that escape the blading are accompanied by shocks of the same family and intensity. It is only required that the air far ahead and behind the blading reach the same value of free-stream pressure while properly satisfying the equations of mass flow, energy, and momentum to account for power and thrust.

It is a necessary condition that the air approaching the propeller annulus acquire no net rotation or angular momentum. The average flow at stations 0 and L.E. must therefore be axial; thus, the angular momentum of the incoming air relative to the cascade is interpreted as being a result of the transverse velocity of the cascade. This transverse

velocity is then directly a measure of the rotational velocity of the corresponding propeller annulus and, in combination with the transverse component of the forces on the airfoils in cascade, determines the power input.

The average flow at station T.E. will be nonaxial with an average tangential velocity component to account for the rotational momentum acquired as a result of the torque on the blades.

It is interesting to note that the average flow conditions at stations L.E. and T.E. are exactly comparable to the conditions at station 1 and station 2, respectively, immediately ahead of and immediately behind, the actuator disk of the conventional momentum theory. Of particular importance is the fact that, under certain conditions (see the cascade patterns of figs. 5(b), 6, and 7), power is absorbed and thrust produced through a pressure drop, rather than a pressure rise, from stations 1 to 2. The changes to the simple momentum theory of reference 1 in order to extend it through the sonic speed range are thereby indicated.

The manner in which the flow proceeds from station T.E. to station 3 far downstream is governed by the average axial Mach number at station T.E.

Whenever the axial Mach number at station T.E. is subsonic (see the cascade patterns of figs. 3, 4, and 5(a)), the axial flow component behaves as a simple subsonic jet in which the trailing-edge conditions are influenced by the adjustment process in the wake. As a result, only one set of free-stream conditions is consistent with an assumed cascade flow pattern.

Whenever the average axial Mach number at station T.E. is supersonic (see the cascade patterns of figs. 5(b), 6, and 7), the flow downstream can exert no influence on the blades whatsoever and hence may be disregarded in the calculations. The flow, however, must behave as a supersonic jet with the accompanying shock and expansion patterns that allow adjustment to higher downstream pressures. A detailed discussion of a similar process is given in reference 3, page 172.

Examination of the general shock and expansion pattern about the airfoils in cascade reveals that supersonic axial flow at station T.E. is possible only when the axial Mach number at station L.E. is sonic or greater. Although sonic inflow is a necessary condition, it is not a sufficient condition for the existence of the supersonic jet. Over a limited range apparently determined by losses such as blade drag introduced into the system, sonic inflow with a subsonic jet may exist (see fig. 5(a)).

COMPUTATIONAL PROCEDURE

The mathematical solutions are relatively straightforward. At certain points, however, difficulties arise which must be resolved by reference to the physical aspects of the problem. In order to discuss these situations the mechanics of the method are briefly described.

It is necessary to develop solutions in an inverse manner; that is, a local flow pattern about the cascade is first determined and then stream conditions consistent with this pattern are determined. The steps in obtaining solutions are as follows:

A. Determination of initial conditions.— The cascade geometry is fixed (see fig. 1) by the propeller annulus under study so that

$$S = b/\sigma$$

$$\beta_C = \beta_X$$

Values of M_R , p_R , ρ_R and, when necessary, either α_R or p_R' , or both, are next chosen. These values should be appropriate to the desired flight and power conditions. In order to obtain first approximations (exact only for supersonic flight) it is suggested that the following relationships be used:

$$\phi_R = \tan^{-1} \frac{J}{\pi x}$$

$$M_R = \frac{M_0}{\sin \phi_R}$$

$$p_R = p_0$$

$$\rho_R = \rho_0$$

The quantities α_R and p_R' are functions primarily of the power and a satisfactory first approximation cannot be given. An exact solution satisfying the desired flight and power conditions requires successive approximations.

B. Local flow-field solution.- The flow pattern about the blades in cascade operating under the assumed initial conditions is calculated using the Prandtl-Meyer relationships of reference 4 and the oblique-shock relationship of reference 5. Second-order effects of entropy losses across the shocks and shock curvature are neglected, and it is assumed that shocks and expansions of opposite families intersect without change in direction. Under these assumptions, it will be found that the flow pattern about the blades in cascade is consistent.

By use of this repeating pattern, the blade forces and local pressures, velocities, and densities may be obtained.

C. Average conditions at station L.E.- Average conditions at station L.E. may be obtained by means of the following equations

$\left(\int_S \right)$ indicates that integrations are taken over one blade spacing at the blade L.E.):

for continuity,

$$\begin{aligned} m_{LE} &= \int_S \rho W \sin \phi \, dS \\ &= (\rho_{LE} W_{LE} \sin \phi_{LE}) S \end{aligned}$$

for axial momentum,

$$\int_S (\rho W^2 \sin^2 \phi + p) \, dS = (\rho_{LE} W_{LE}^2 \sin^2 \phi_{LE} + p_{LE}) S$$

for transverse momentum,

$$\int_S \rho W^2 \sin \phi \cos \phi \, dS = (\rho_{LE} W_{LE}^2 \sin \phi_{LE} \cos \phi_{LE}) S$$

and, for energy (Energy must be conserved; thus the reference conditions are taken as a convenient sample),

$$\frac{\gamma}{\gamma - 1} \frac{p_R}{\rho_R} + \frac{W_R^2}{2} = \frac{\gamma}{\gamma - 1} \frac{p_{LE}}{\rho_{LE}} + \frac{W_{LE}^2}{2}$$

The preceding equations when solved simultaneously lead to two sets of average conditions corresponding to either a high or a low value of p_{LE} . In every case, the high-pressure solution is selected. The low-pressure solution, corresponding to a supersonic axial Mach number, requires a physically impossible throat ahead of the blades.

The average axial velocity of the slipstream at station L.E. and the transverse velocity of the cascade may be determined by

$$(V_{LE})_a = W_{LE} \sin \phi_{LE}$$

$$U = W_{LE} \cos \phi_{LE}$$

the rotational speed of the corresponding propeller annulus in revolutions per second becomes

$$n = \frac{U}{2\pi r}$$

and the average axial Mach number at station L.E. is

$$(M_{LE})_a = \frac{(V_{LE})_a}{\sqrt{\frac{\gamma p_{LE}}{\rho_{LE}}}}$$

D. Average conditions at station T.E.- Average conditions at station T.E. may be determined in a manner similar to that used in finding the conditions at station L.E. A simpler method, however, is as follows:

For continuity,

$$(\rho_{TE} W_{TE} \sin \phi_{TE}) S = m_{LE}$$

for axial momentum,

$$\left[(\rho_{TE} W_{TE}^2 \sin^2 \phi_{TE} + p_{TE}) - (\rho_{LE} W_{LE}^2 \sin^2 \phi_{LE} + p_{LE}) \right] S = F \cos \beta_c$$

for transverse momentum,

$$(\rho_{LE} W_{LE}^2 \sin \phi_{LE} \cos \phi_{LE} - \rho_{TE} W_{TE}^2 \sin \phi_{TE} \cos \phi_{TE}) S = F \sin \beta_c$$

and, for energy,

$$\frac{\gamma}{\gamma - 1} \frac{p_{TE}}{\rho_{TE}} + \frac{W_{TE}^2}{2} = \frac{\gamma}{\gamma - 1} \frac{p_R}{\rho_R} + \frac{W_R^2}{2}$$

As before, when the preceding equations are solved simultaneously both a high- and a low-pressure solution are mathematically possible. The choice of solution is determined by examination of the flow pattern. When a shock exists in the blading, the high-pressure solution is selected and, when no shock exists, the low-pressure solution applies.

The axial and transverse components of the average slipstream velocity at station T.E. are now found to be

$$(V_{TE})_a = W_{TE} \sin \phi_{TE}$$

$$(V_{TE})_t = U - W_{TE} \cos \phi_{TE}$$

and the average axial Mach number at station T.E. is

$$(M_{TE})_a = \frac{(V_{TE})_a}{\sqrt{\frac{\gamma p_{TE}}{\rho_{TE}}}}$$

The torque force per blade per unit radial distance of the propeller annulus is now

$$\frac{Q}{r} = m(V_{TE})_t = F \sin \beta_x$$

The power per unit mass flow is

$$\frac{P}{\dot{m}} = \frac{U}{\dot{m}} \frac{Q}{r} = \frac{2\pi n r}{\dot{m}} \frac{Q}{r}$$

Thrust per unit mass flow is equal to

$$\frac{T}{\dot{m}} = \frac{F}{\dot{m}} \cos \beta_x$$

E. Determination of flight conditions.- For the subsonic average axial Mach number at station T.E., the following relationships must be simultaneously satisfied:

$$p_0 = p_3$$

$$\frac{p_{LE}}{\rho_{LE}^\gamma} = \frac{p_0}{\rho_0^\gamma}$$

$$\frac{p_{TE}}{\rho_{TE}^\gamma} = \frac{p_3}{\rho_3^\gamma}$$

$$\frac{P}{\dot{m}} + \frac{\gamma}{\gamma - 1} \frac{p_{LE}}{\rho_{LE}} + \frac{(v_{LE})_a^2}{2} = \frac{\gamma}{\gamma - 1} \frac{p_3}{\rho_3} + \frac{v_3^2}{2}$$

$$\frac{T}{\dot{m}} = (v_{3a} - v_0)$$

where

$$v_{3a} = \left[v_3^2 - (v_{3t})^2 \right]^{1/2}$$

and, with the assumption of constant mean flow radius

$$v_{3t} = (v_{TE})_t$$

For the supersonic average axial Mach number at station T.E., the conditions are as follows:

For supersonic Mach number at station L.E.,

$$p_0 = p_{LE}$$

$$\rho_0 = \rho_{LE}$$

$$V_0 = V_{LE}$$

and, for sonic axial Mach number at station L.E., some freedom in the choice of free-stream pressure exists. The lower limit of free-stream pressure is p_{LE} , corresponding to sonic flight Mach number. The upper limit is determined by the condition that it must not be too high to prevent the exit flow. Thus, if the air at the trailing edges must adjust to free-stream pressure directly (under the assumption of independence of annulus operation), this upper limit is determined by the pressure rise through a normal shock at the trailing edges.

After ρ_0 has been selected, it follows directly that

$$\rho_0 = \rho_{LE} \left(\frac{p_0}{p_{LE}} \right)^{1/\gamma}$$

and

$$V_0 = \left[\frac{2\gamma}{\gamma - 1} \left(\frac{p_{LE}}{\rho_{LE}} - \frac{p_0}{\rho_0} \right) + (V_{LE})_a^2 \right]^{1/2}$$

For all these flight conditions the efficiency is

$$\eta = \frac{(T/m)V_0}{P/m}$$

REPRESENTATIVE SOLUTIONS

Three representative solutions are presented in table I. They illustrate, respectively, subsonic forward speed with subsonic inflow, subsonic forward speed with sonic inflow, and supersonic forward speed.

No attempt has been made to connect the solutions presented to any specific propeller, although it is believed that the conditions investigated lie within the range of interest for supersonic propellers. The solutions are presented primarily to show that operation throughout the Mach number range may be studied.

When forward speed and inflow velocity are both subsonic, the variations in Mach number, velocity, pressure, and density follow the general trends established by the simple axial-momentum theory of reference 1. A specific case solved by both methods is presented in table II for a power disk loading coefficient P_c of 0.224 and a flight Mach number of 0.70.

The minor differences present are due largely to consideration in the present theory of drag due to lift associated with supersonic airfoils. Losses associated with this drag due to lift that were not present in the theory of reference 1 cause the reduction in efficiency from 0.974 to 0.883.

With subsonic forward speed and sonic inflow the average Mach number at station T.E. may be subsonic or supersonic depending on the power. In the second example of table I, the forward Mach number is high enough and the power input is low enough so that supersonic conditions prevail at station T.E.

When the forward speed is supersonic, as in the third example of table I, the conditions at station T.E. are then supersonic.

It is worth noting that, in each case presented, operation is relatively efficient.

CONCLUDING REMARKS

The cascade-general-momentum theory herein outlined seems adequate in describing the general operation of a supersonic propeller annulus throughout the flight Mach number range. Although some problems concerned with the flow details in certain transition regions remain, they appear to be amenable to further analysis.

Effects such as those due to thickness, shock interference, and viscosity may be handled if the local flow pattern is determinable since a local flow pattern is the only requirement for a solution. The investigation of such factors as tip and shank effects and radial gradients that exist in an actual propeller but are not considered in the present theory will require considerable further development.

In its present form, the solution to any particular problem usually involves successive approximation. The development of a more direct method is needed. Even in its present form, however, the method should be useful in establishing the general trends of propeller operation throughout the transonic and supersonic flight regions.

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TABLE I

THREE REPRESENTATIVE SOLUTIONS TO THE CASCADE—GENERAL-MOMENTUM THEORY
FOR THE OPERATION OF A SUPERSONIC PROPELLER ANNULUS

Assumed Initial Conditions			
	I Subsonic forward speed, subsonic inflow	II Subsonic forward speed, sonic inflow	III Supersonic forward speed
P_R	391.8	373.8	391.8
$P_{R'}$	unnecessary for these cases		
ρ_R	.000586	.000566	.000586
M_R	1.330	1.518	1.649
β_c	34.75	43.30	49.70
a_R	3.00	2.05	3.00
S	6.667	6.667	6.667

Calculated Average Flow Conditions Throughout Slipstream				
	Station 0	Leading edge	Trailing edge	3
Solution I				
p	384.4	369.3	399.2	384.4
V	679.0	716.9	679.1	715.5
ρ	.000578	.000562	.000593	.000577
M	.704	.747	.700	.714
Solution II				
p	376.6	373.8	338.6	376.6
V	956.2	961.2	1031.6	-----
ρ	.000569	.000566	.000528	-----
M	.994	1.000	1.088	Subsonic
Solution III				
p	391.8	391.8	370.2	391.8
V	1160.0	1160.0	1210.3	-----
ρ	.000586	.000586	.000562	-----
M	1.200	1.200	1.260	Supersonic

Resultant Operating Conditions			
	I	II	III
β_x	34.75	43.30	49.70
β_0	31.55	41.17	46.70
P/S	20.20	5.87	27.15
η	.885	.926	.900
Local cascade flow pattern	fig. 3	fig. 5(b)	fig. 7

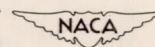
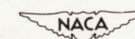


TABLE II

COMPARISON OF SOLUTION OBTAINED BY METHOD OF REFERENCE 1 AND PRESENT THEORY
 (SOLUTION I, TABLE I) FOR A POWER-DISK LOADING COEFFICIENT P_c
 OF 0.224 AND A FLIGHT MACH NUMBER OF 0.70

Station	0		1 or L.E.		2 or T.E.		3	
Method	Reference 1	Present	Reference 1	Present	Reference 1	Present	Reference 1	Present
Mach number ratio, M/M_0	1.000	1.000	1.069	1.061	0.992	0.994	1.055	1.053
Velocity ratio, V/V_0	1.000	1.000	1.065	1.056	0.998	1.000	1.058	1.054
Pressure ratio, p/p_0	1.000	1.000	0.956	0.961	1.042	1.039	1.000	1.000
Density ratio, ρ/ρ_0	1.000	1.000	0.969	0.972	1.030	1.026	1.000	0.998

Method	Reference 1	Present
η	0.974	0.885
P/S, hp/sq ft	20.65	20.20



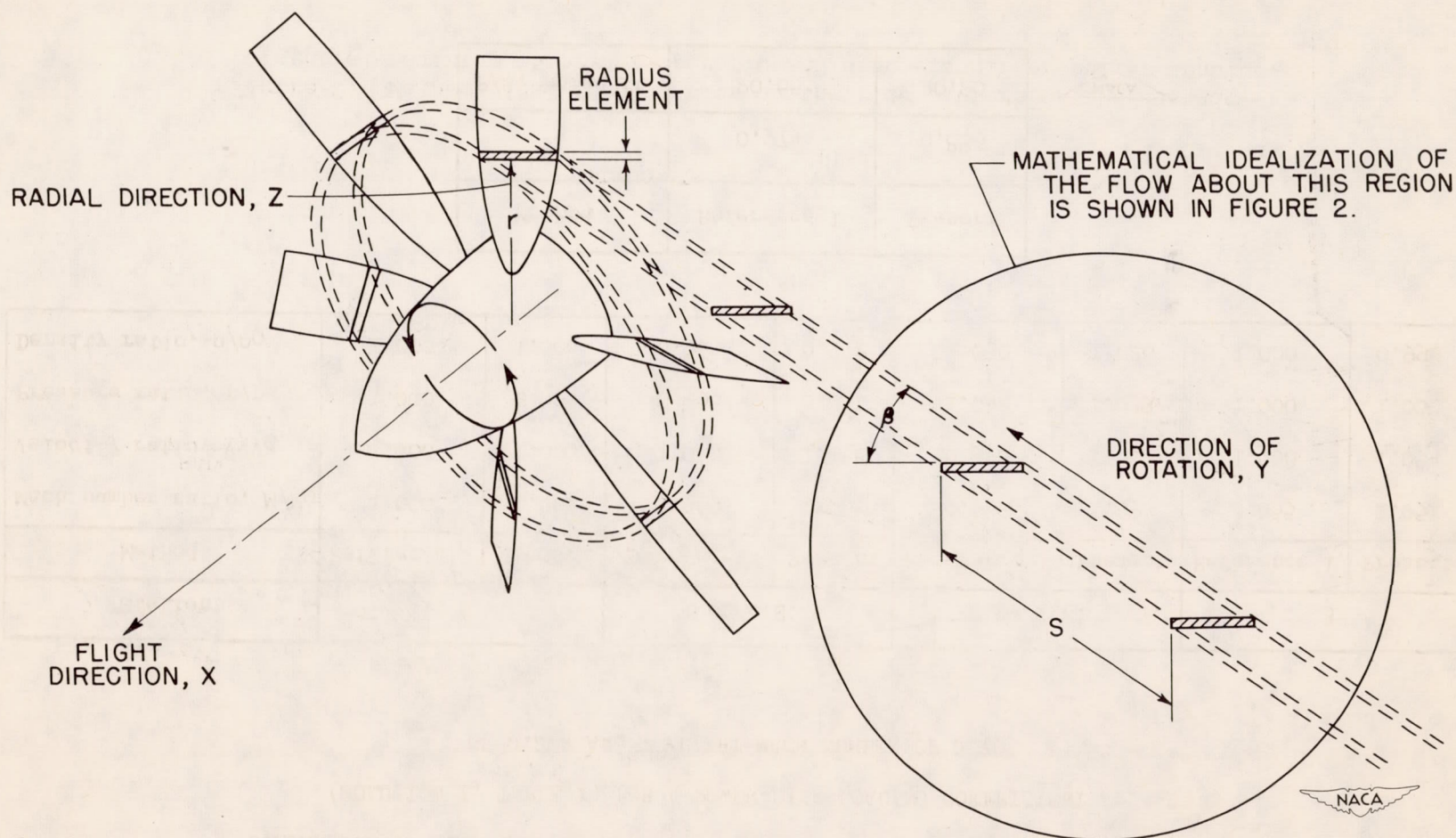


Figure 1.- Generation of a two-dimensional cascade to represent a propeller annulus.

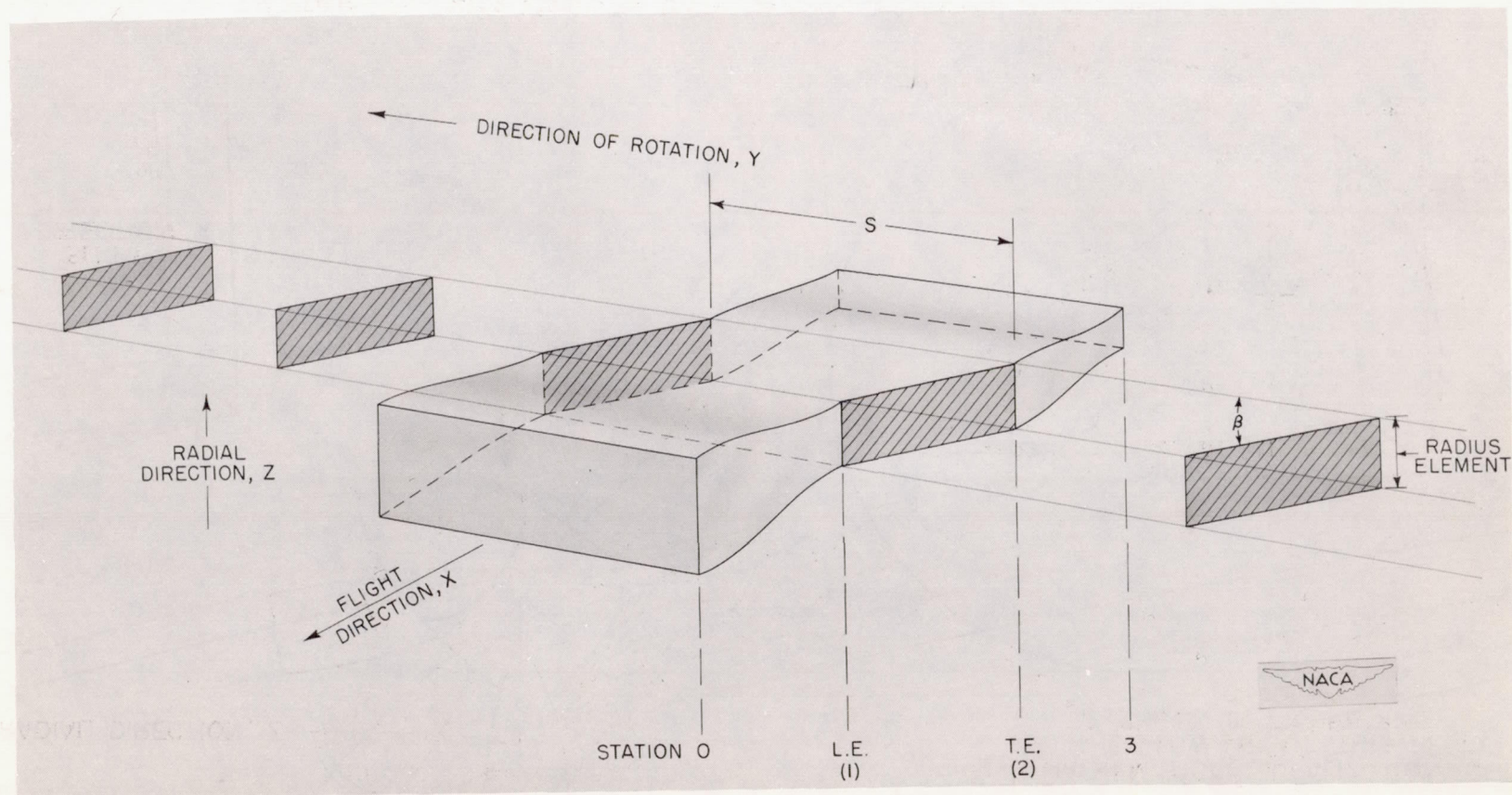


Figure 2.- Mathematical idealization of the flow pattern of the cascade—
general-momentum theory of operation of a supersonic propeller annulus.

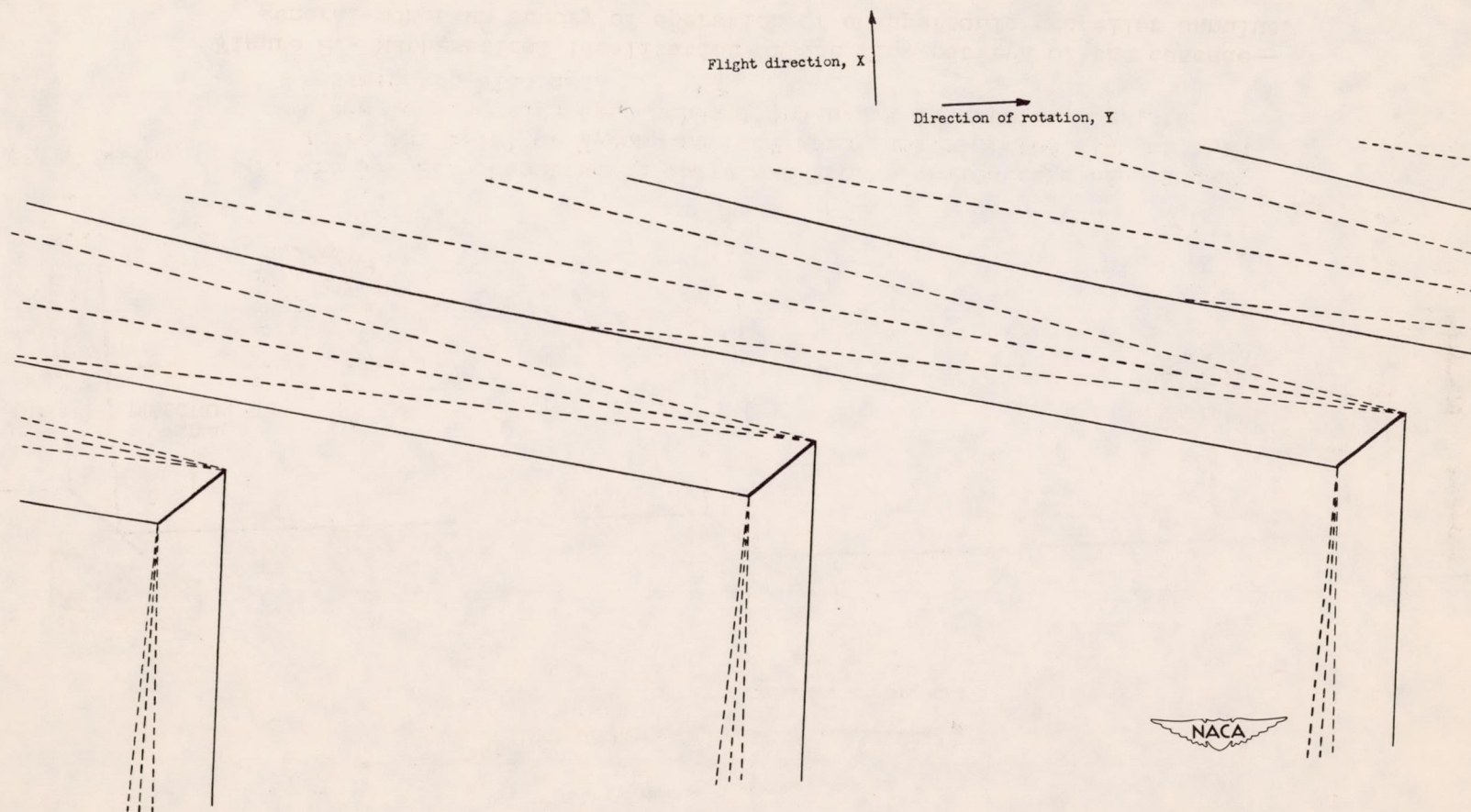


Figure 3.- Flow about an infinite two-dimensional supersonic cascade where the axial or X-component of Mach number of the flow at the blade leading edge is sufficiently subsonic so that no interference exists between the blading.

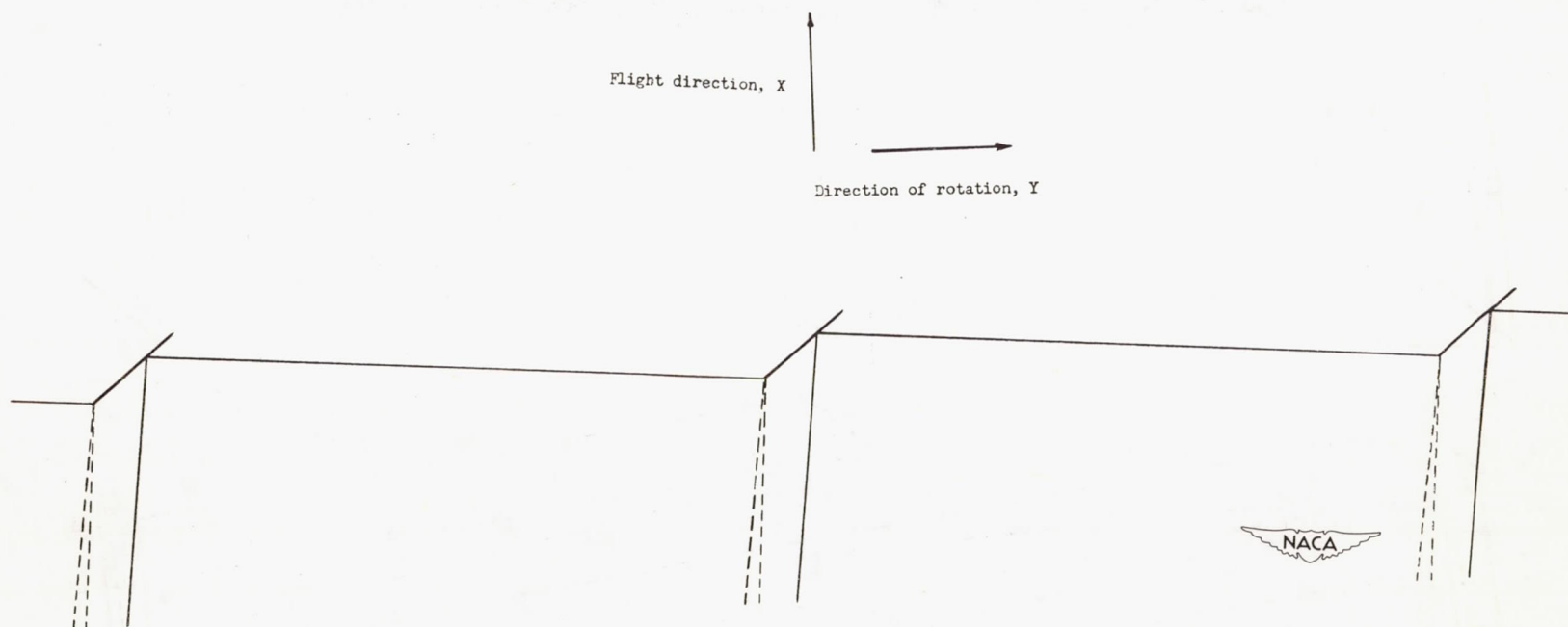
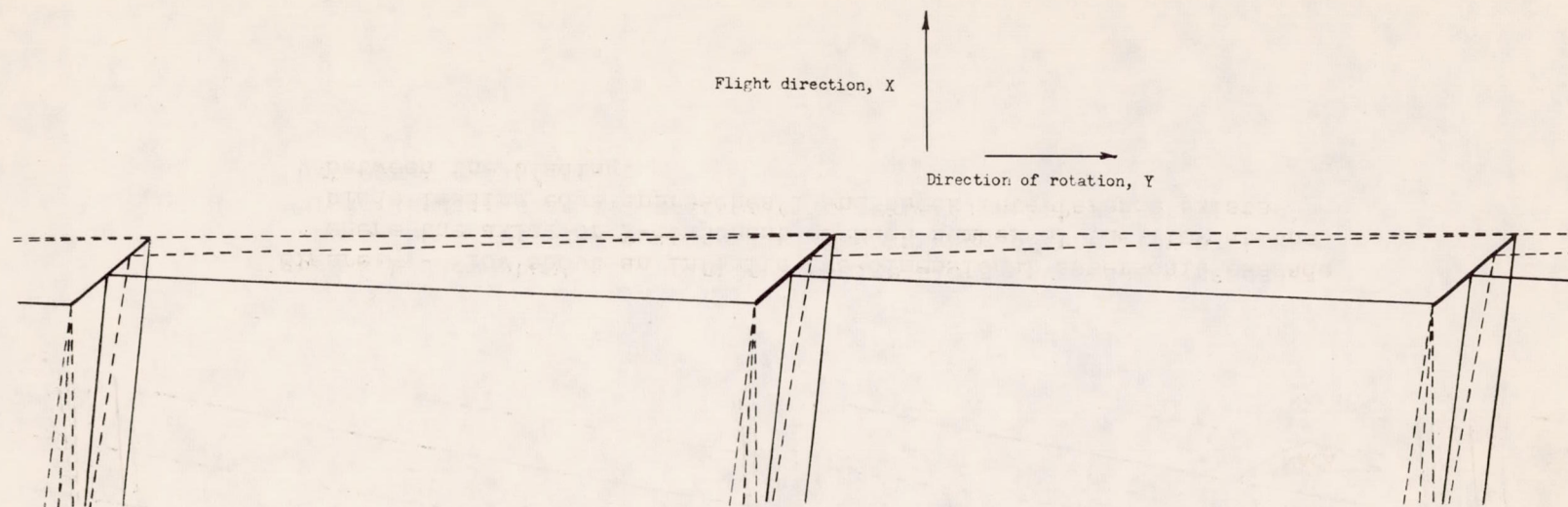
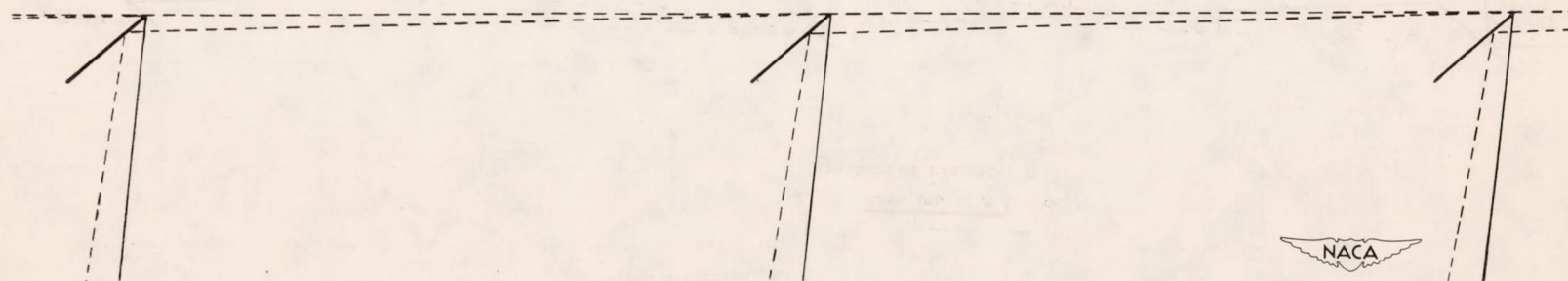


Figure 4.- Flow about an infinite two-dimensional supersonic cascade where the axial or X-component of Mach number of the flow at the blade leading edge approaches 1 and shock interference exists between the blading.



(a) Pattern when pressure immediately behind the trailing edge is higher than the pressure at the leading edge.



(b) Pattern for pressure immediately behind the trailing edge equal to pressure on the upper surface of the blade.

Figure 5.- Flow about an infinite two-dimensional supersonic cascade where the axial or X-component of Mach number of the flow at the blade leading edge is 1 and shock-expansion interference exists between the blading.

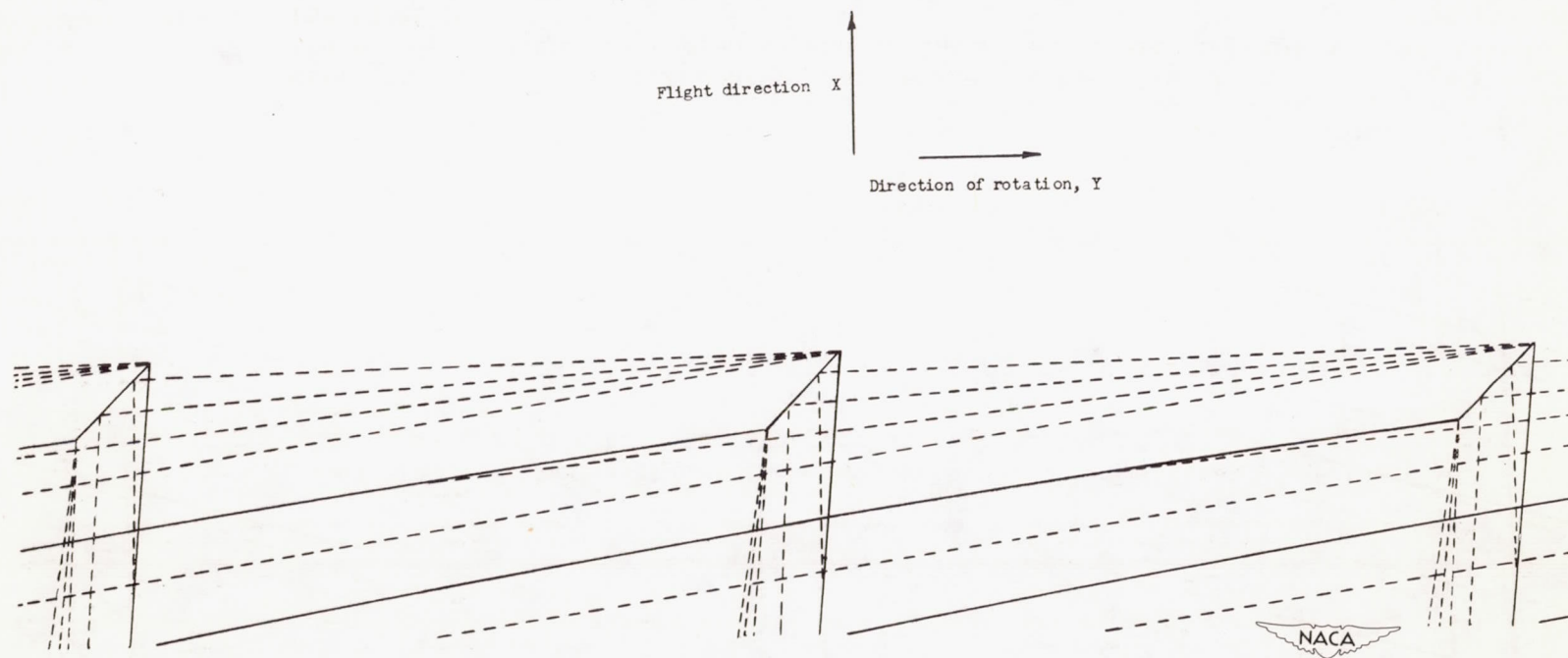


Figure 6.- Flow about an infinite two-dimensional supersonic cascade where the axial or X-component of Mach number of the flow at the blade leading edge is supersonic and expansion interference exists between the blading.

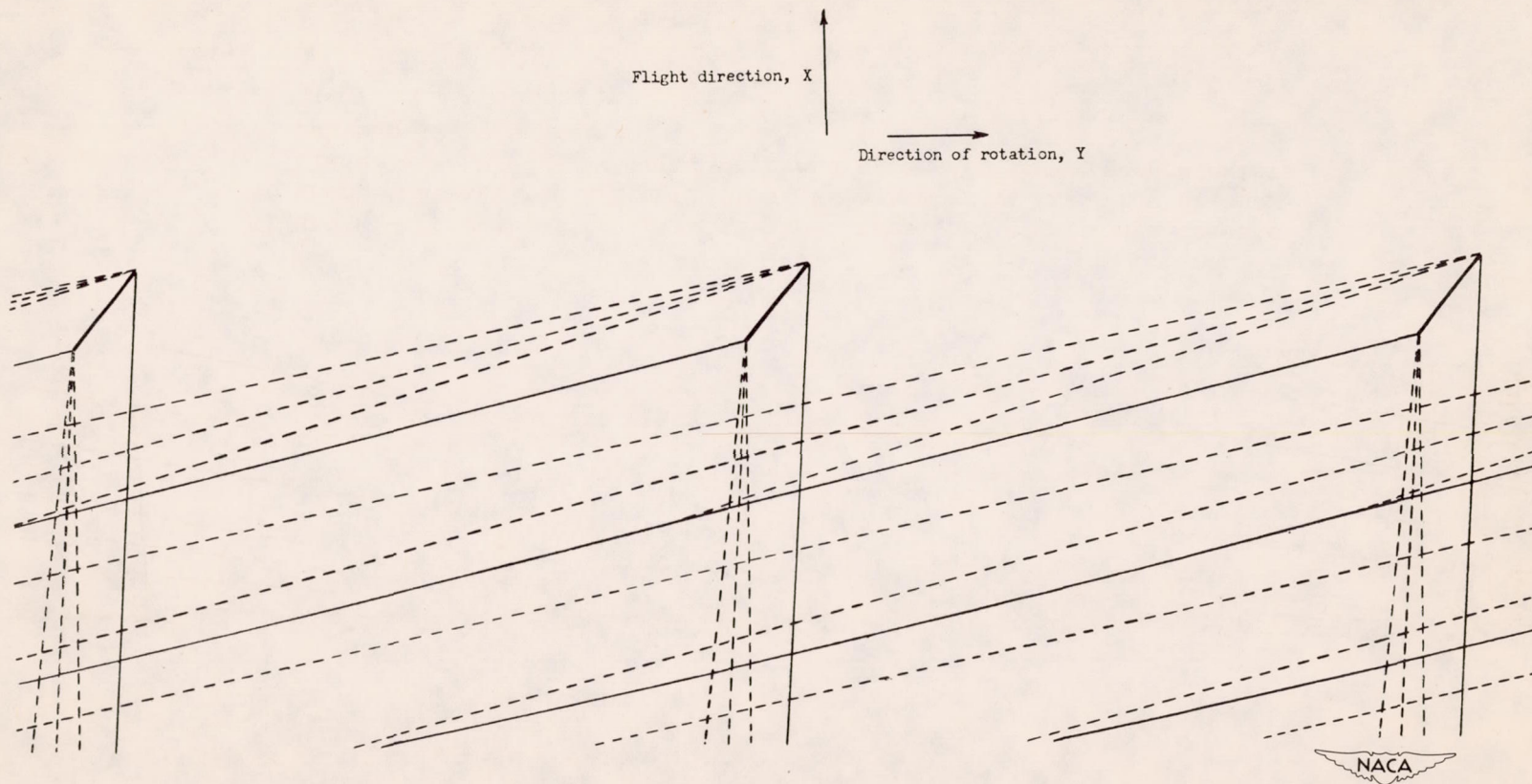


Figure 7.- Flow about an infinite two-dimensional supersonic cascade where the axial or X-component of Mach number of the flow at the blade leading edge is supersonic and no interference exists between the blading.